General Aspects of Lorentz-violating Theories

Dra. Letícia Lisboa dos Santos

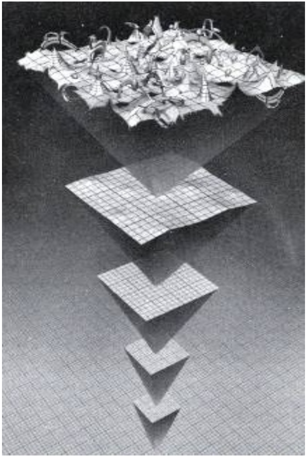
PPG - Física Universidade Federal do Maranhão

November 2, 2022

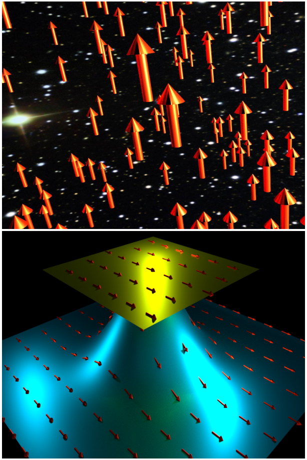
Outline

- Motivation for LV-Theories
- Nonminimal SME
- Propagator technique: Maxwell-Chern-Simons-Proca
- Podolsky's electrodynamics
- higher derivative CPT-even LV term
- Maxwell electrodynamics modified by a CPT-odd dimension-5 higher-derivative term
- Non-minimal Planar Electrodynamics Modified by LV-terms

- It is believed that the nature of space-time at the Planck scale $(\text{Planck length} \sim 10^{-35})$ is not necessarily continuous;
  [Bernadotte, Klinkhamer (2007), hep-ph/0610216]
- Violation of Lorentz Symmetry from String Theory;
  [Kostelecký, Samuel (1989), Phys. Rev. D **39**,683]
- Loop Quantum Gravity Theories can also generate LV;
  [R. Gambini and J. Pullin (1999) Phys. Rev. D **59**, 124021 , gr-qc/9809038]
- Discrete symmetry violation CPT $(\text{CPT}$ leads to LV.)



- The Lorentz symmetry break can be incorporated in two ways:
  - Explicit breaking:
    - Contraction between Background Fields and Field Operators;
    - Flat Space.
  - Spontaneous Break:
    - Dynamic Background Fields → Symmetry Break.
    - Curved space.
- Representation of the both types Background fields.



SME

$$\text{The SME foton sector}\rangle = \begin{cases} \text{CPT-even}\rangle & \text{Kostelecký} \\ \text{CPT-odd}\rangle & \text{Carrol-Field-Jackiw} \end{cases}$$

- The Lagrange density is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}A_\lambda \left(\hat{k}_{AF}\right)_\kappa F_{\mu\nu} - \frac{1}{2}F_{\kappa\lambda}\left(\hat{k}_F\right)^{\kappa\lambda\mu\nu} F_{\mu\nu}. \quad (1)$$

- where the operators $\hat{k}_{AF}$ and $\hat{k}_F$ have the form

$$\left(\hat{k}_{AF}\right)_\kappa = \sum_{d\ \geq 3,\ \text{odd}} \left(k_{AF}^{(d)}\right)_\kappa^{\alpha_1 \ldots \alpha_{(d-3)}} \partial_{\alpha_1} \ldots \partial_{\ldots\alpha_{(d-3)}}, \quad (2)$$

$$\left(\hat{k}_F\right)^{\kappa\lambda\mu\nu} = \sum_{d\ \geq 4,\ \text{even}} \left(k_F^{(d)}\right)^{\kappa\lambda\mu\nu\alpha_1 \ldots \alpha_{(d-4)}} \partial_{\alpha_1} \ldots \partial_{\ldots\alpha_{(d-4)}}. \quad (3)$$

- $d = 3, 4$ the minimal version of the SME;
- $d \geq 5$ non-minimal version (non-renormalizable operators).

-
- The Lagrangean of the Maxwell-Chern-Simons-Proca theory is shown below

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{2}\varepsilon^{\mu\nu\rho}A_{\mu}\partial_{\nu}A_{\rho} + \frac{1}{2}m^{2}A_{\mu}A^{\mu}. \tag{4}$$

- From the density above, it's possible to calculate the propagator;
- To do that, we will use the method of "squaring" the Lagrangean density. Next, we will show how this method works.

The first step is to rewrite the Lagrangean density in its bilinear form, namely:

$$L = A_\mu \hat{\Theta}^{\mu\nu} A_\nu. \tag{5}$$

We begin by rewriting all the terms of the Lagrangeana as a function of the gauge field A_α, namely:

$$\begin{aligned} L &= -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) + \frac{\theta}{2}\varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + \frac{1}{2} m^2 A_\mu A^\mu, \\ L &= -\frac{1}{4}\left(2\partial_\mu A_\nu \partial^\mu A^\nu - 2\partial_\mu A_\nu \partial^\nu A^\mu\right) + A_\mu \left(\frac{\theta}{2}\varepsilon^{\mu\rho\nu}\partial_\rho + \frac{1}{2} m^2 g^{\mu\nu}\right) A_\nu, \\ L &= -\frac{1}{2}\left(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu\right) + A_\mu \left(\frac{\theta}{2}\varepsilon^{\mu\rho\nu}\partial_\rho + \frac{1}{2} m^2 g^{\mu\nu}\right) A_\nu, \end{aligned} \tag{6}$$

Using the identities bellow

$$\partial_\mu A_\nu \partial^\mu A^\nu = \partial_\mu \left(A_\nu \partial^\mu A^\nu \right) - A_\nu \partial_\mu \partial^\mu A^\nu, \tag{7}$$

$$\partial_\mu A_\nu \partial^\nu A^\mu = \partial_\mu \left(A_\nu \partial^\nu A^\mu \right) - A_\nu \partial_\mu \partial^\nu A^\mu, \tag{8}$$

we get:

$$L = \frac{1}{2} A_\mu \left(\partial_\alpha \partial^\alpha g^{\mu\nu} - g^{\mu\lambda} g^{\alpha\nu} \partial_\alpha \partial_\lambda + \theta \varepsilon^{\mu\rho\nu} \partial_\rho + m^2 g^{\mu\nu} \right) A_\nu + \underbrace{\partial_\mu S^\mu}_{4-\text{divergence}} ,$$

where we define $S^\mu \equiv -\frac{1}{2} \left(A_\nu \partial^\mu A^\nu - A_\nu \partial^\nu A^\mu \right)$

Then, we get the following form for the operator $\hat{\Theta}^{\mu\nu}$, namely:

$$\hat{\Theta}^{\mu\nu} \equiv \partial_\alpha \partial^\alpha g^{\mu\nu} - g^{\mu\lambda} g^{\alpha\nu} \partial_\alpha \partial_\lambda + \theta \varepsilon^{\mu\rho\nu} \partial_\rho + m^2 g^{\mu\nu}, \tag{9}$$

or we can rewrite it as

$$\hat{\Theta}^{\mu\nu} = \partial_\alpha \partial^\alpha g^{\mu\nu} - \partial^\mu \partial^\nu + \theta \varepsilon^{\mu\rho\nu} \partial_\rho + m^2 g^{\mu\nu}. \tag{10}$$

In the momentum space the equation above becomes:

$$\hat{\Theta}^{\mu\nu} = -p_\alpha p^\alpha g^{\mu\nu} + p^\mu p^\nu - \mathrm{i}\theta \varepsilon^{\mu\rho\nu} p_\rho + m^2 g^{\mu\nu}. \tag{11}$$

Let us now define the following projectors:

$$\begin{cases} \hat{\theta}^{\mu\nu} \equiv g^{\mu\nu} - \omega^{\mu\nu} \rightarrow \text{Transverse projector} \\ \hat{\omega}^{\mu\nu} \equiv \frac{p^\mu p^\nu}{p^2} \rightarrow \text{Longitudinal projector} \\ \hat{S}^{\mu\nu} \equiv -\mathrm{i}\varepsilon^{\mu\rho\nu} p_\rho \rightarrow \text{Chern-Simons projector} \end{cases}. \tag{12}$$

The above projectors satisfy the following properties:

	$\hat{\theta}_{\alpha\nu}$	$\hat{\omega}_{\alpha\nu}$	$\hat{S}_{\alpha\nu}$
$\hat{\theta}^{\mu\alpha}$	$\hat{\theta}^{\mu}{}_{\nu}$	0	$\hat{S}^{\mu}{}_{\nu}$
$\hat{\omega}^{\mu\alpha}$	0	$\hat{\omega}^{\mu}{}_{\nu}$	0
$\hat{S}^{\mu\alpha}$	$\hat{S}^{\mu}{}_{\nu}$	0	$p^2\hat{\theta}^{\mu}{}_{\nu}$

Table: Multiplicative operator algebra fulfilled by $\hat{\theta}^{\mu\nu}$, $\hat{\omega}^{\mu\nu}$ and $\hat{S}^{\mu\nu}$. The products are supposed to be in the ordering "column times row."

We can rewrite the operator $\hat{\Theta}^{\mu}$ in terms of the projectors defined in the previous table, that is:

$$
\begin{aligned}
\hat{\Theta}^{\mu\nu} &= \left(-p^2 + m^2\right) g^{\mu\nu} + p^{\mu}p^{\nu} - \mathrm{i}\theta\varepsilon^{\mu\rho\nu}p_{\rho}, \\
\hat{\Theta}^{\mu\nu} &= \left(-p^2 + m^2\right) g^{\mu\nu} + \left(p^2 - m^2\right)\frac{p^{\mu}p^{\nu}}{p^2} + m^2\frac{p^{\mu}p^{\nu}}{p^2} + \theta\hat{S}^{\mu\nu}, \\
\hat{\Theta}^{\mu\nu} &= \left(-p^2 + m^2\right)\left(g^{\mu\nu} - \frac{p^{\mu}p^{\nu}}{p^2}\right) + m^2\hat{\omega}^{\mu\nu} + \theta\hat{S}^{\mu\nu}, \\
\hat{\Theta}^{\mu\nu} &= \left(-p^2 + m^2\right)\hat{\theta}^{\mu\nu} + m^2\hat{\omega}^{\mu\nu} + \theta\hat{S}^{\mu\nu}.
\end{aligned} \tag{13}
$$

We know that the propagator is given by:

$$
\hat{\Lambda}^{\mu\nu} \equiv \left(\hat{\Theta}^{\mu\nu}\right)^{-1}. \tag{14}
$$

Therefore, we need to invert the operator $\hat{\Theta}^{\mu}$ in order to get the propagator.

To accomplish our goal, we intend to use the identity given below to derive the propagator, namely:

$$\hat{\Lambda}_{\mu\alpha}\hat{\Theta}^{\alpha\nu} = \delta^{\mu}_{\nu}. \tag{15}$$

To proceed using the above identity, we will propose the following structure for the $Lam\hat{b}da_{\mu\alpha}$, namely:

$$\hat{\Lambda}_{\mu\alpha} \equiv a_{\hat{\theta}}\hat{\theta}_{\mu\alpha} + b_{\hat{\omega}}\hat{\omega}_{\mu\alpha} + c_{\hat{S}}\hat{S}_{\mu\alpha}. \tag{16}$$

We now need to find the value of the parameters $\left\{a_{\hat{\theta}}, b_{\hat{\omega}}, c_{\hat{S}}\right\}$ in order to find the propagator and, to do this, we will use the identity (15) together with the results shown in the previous table. Thus, we have:

$$\hat{\Lambda}_{\mu\alpha}\hat{\Theta}^{\alpha\nu} = \delta^{\nu}_{\mu},$$

$$\left(a_{\hat{\theta}}\hat{\theta}_{\mu\alpha} + b_{\hat{\omega}}\hat{\omega}_{\mu\alpha} + c_{\hat{S}}\hat{S}_{\mu\alpha}\right)\left(\left(-p^2 + m^2\right)\hat{\theta}^{\alpha\nu} + m^2\hat{\omega}^{\alpha\nu} + \theta\hat{S}^{\alpha\nu}\right) = \delta^{\nu}_{\mu}.$$

By distributing the above terms, we get:

$$\begin{aligned}\delta_{\mu}^{\nu} = & a_{\hat{\theta}}\left(-p^2+m^2\right)\hat{\theta}_{\mu\alpha}\hat{\theta}^{\alpha\nu} + a_{\hat{\theta}} m^2 \hat{\theta}_{\mu\alpha}\hat{\omega}^{\alpha\nu} + a_{\hat{\theta}}\theta\hat{\theta}_{\mu\alpha}\hat{S}^{\alpha\nu} \\ & + b_{\hat{\omega}}\left(-p^2+m^2\right)\hat{\omega}_{\mu\alpha}\hat{\theta}^{\alpha\nu} + b_{\hat{\omega}} m^2 \hat{\omega}_{\mu\alpha}\hat{\omega}^{\alpha\nu} + b_{\hat{\omega}}\theta\hat{\omega}_{\mu\alpha}\hat{S}^{\alpha\nu} \\ & + c_{\hat{S}}\left(-p^2+m^2\right)\hat{S}_{\mu\alpha}\hat{\theta}^{\alpha\nu} + c_{\hat{S}} m^2 \hat{S}_{\mu\alpha}\hat{\omega}^{\alpha\nu} + c_{\hat{S}}\theta\hat{S}_{\mu\alpha}\hat{S}^{\alpha\nu},\end{aligned}$$

and now applying the results for the projectors,

	$\hat{\theta}_{\alpha\nu}$	$\hat{\omega}_{\alpha\nu}$	$\hat{S}_{\alpha\nu}$
$\hat{\theta}^{\mu\alpha}$	$\hat{\theta}^{\mu}{}_{\nu}$	0	$\hat{S}^{\mu}{}_{\nu}$
$\hat{\omega}^{\mu\alpha}$	0	$\hat{\omega}^{\mu}{}_{\nu}$	0
$\hat{S}^{\mu\alpha}$	$\hat{S}^{\mu}{}_{\nu}$	0	$p^2\hat{\theta}^{\mu}{}_{\nu}$

we finally get

$$\begin{aligned}\delta_{\nu}^{\mu} = & \left[a_{\hat{\theta}}\left(-p^2+m^2\right) + c_{\hat{S}}\theta p^2\right]\hat{\theta}_{\mu}{}^{\nu} \\ & + b_{\hat{\omega}} m^2 \hat{\omega}_{\mu}{}^{\nu} + \left[a_{\hat{\theta}}\theta + c_{\hat{S}}\left(-p^2+m^2\right)\right]\hat{S}_{\mu}{}^{\nu}. \end{aligned} \tag{17}$$

From the equation above, we can obtain a set of equations for the parameters $\left\{a_{\hat{\theta}}, b_{\hat{\omega}}, c_{\hat{S}}\right\}$ taking into account both the linear independence of the basis $\left\{\hat{\theta}^{\mu\nu},\, \hat{\omega}^{\mu\nu},\, \hat{S}^{\mu\nu}\right\}$ and the results shown in the projector's table as follows:

$$1^{o}) \rightarrow \left\{ \begin{array}{c} a_{\hat{\theta}}\left(-p^{2}+m^{2}\right)+c_{\hat{S}}\theta p^{2}=1 \\ a_{\hat{\theta}}\theta+c_{\hat{S}}\left(-p^{2}+m^{2}\right)=0 \end{array} \right. , \tag{18}$$

$$2^{o}) \rightarrow b_{\hat{\omega}}m^{2}=1, \tag{19}$$

$$3^{o}) \rightarrow \left\{ \begin{array}{c} a_{\hat{\theta}}\left(-p^{2}+m^{2}\right)+c_{\hat{S}}\theta p^{2}=1 \\ \left[a_{\hat{\theta}}\theta+c_{\hat{S}}\left(-p^{2}+m^{2}\right)\right]p^{2}=0 \end{array} \right. . \tag{20}$$

Solving the equations above, we have:

$$c_{\hat{S}}=\frac{\theta}{-\left(p^{2}-m^{2}\right)^{2}+\theta^{2}p^{2}},\; a_{\hat{\theta}}=\frac{\left(p^{2}-m^{2}\right)}{-\left(p^{2}-m^{2}\right)^{2}+\theta^{2}p^{2}},\; b_{\hat{\omega}}=\frac{1}{m^{2}}.$$

With the above results, the propagator takes the following form:

$$\hat{\Lambda}^{\mu\nu} = \frac{1}{-\left(p^2-m^2\right)^2+\theta^2 p^2}\left[\left(p^2-m^2\right)g^{\mu\nu}-\mathrm{i}\theta\varepsilon^{\mu\rho\nu}p_\rho - \left(p^2-m^2-\theta^2\right)\frac{p^\mu p^\nu}{m^2}\right]. \tag{21}$$

Using the standard propagator notation, that is,

$$\mathrm{i}D_F^{\mu\nu} = \langle 0|\, TA^\mu(x)\, A^\nu(x')\,|0\rangle\,,$$

we get:

$$\mathrm{i}D_F^{\mu\nu} = \frac{\mathrm{i}}{\left(p^2+E_+ +\mathrm{i}\varepsilon\right)\left(p^2+E_- +\mathrm{i}\varepsilon\right)}\left[\left(p^2-m^2\right)g^{\mu\nu} \tag{22}$$

$$- \left(p^2-m^2-\theta^2\right)\frac{p^\mu p^\nu}{m^2} - \mathrm{i}\theta\varepsilon^{\mu\rho\nu}p_\rho\right], \tag{23}$$

where we made use of the following definitions: $E_\pm \equiv \frac{1}{2}\theta^2 \pm \frac{1}{2}\theta\sqrt{4m^2+\theta^2}+m^2$

We observe that when we perform the limit $\theta \to 0$ we recover the propagator of the Maxwell-Proca theory, that is

$$\mathrm{i}D_F^{\mu\nu} = -\frac{\mathrm{i}}{(p^2 - m^2 + \mathrm{i}\varepsilon)}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2}\right). \tag{24}$$

The poles of the equation (22) gives us the dispersion relations, i.e.

$$-\left(p^2 - m^2\right)^2 + \theta^2 p^2 = 0, \tag{25}$$

whose solutions are

$$\begin{aligned} p^2 &= \frac{1}{2}\theta^2 \pm \frac{1}{2}\theta\sqrt{4m^2+\theta^2} + m^2, \\ \text{or,}\ \left(p^0_{(\pm)}\right)^2 &= \boldsymbol{p}^2 + m^2 + \frac{1}{2}\theta^2 \pm \frac{1}{2}\theta\sqrt{4m^2+\theta^2}. \end{aligned} \tag{26}$$

Podolsky's electrodynamics

The Podolsky's model is represented by the following Lagrangian density

$$\mathcal{L}_{Podolsky} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\theta^2}{2}\partial_\alpha F^{\alpha\beta}\partial_\lambda F^{\lambda}{}_{\beta} + \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2. \qquad (27)$$

In order to calculate the Feynman propagator, we can rewrite the Lagrangian density in its bilinear form, that is:

$$\mathcal{L} = \frac{1}{2}A^\nu O_{\mu\nu}A^\mu, \qquad (28)$$

where $O_{\mu\nu}$ is the following tensor operator

$$O_{\mu\nu} = \left(\Box + \theta^2\Box^2\right)\Theta_{\mu\nu} - \frac{1}{\xi}\Box\Omega_{\mu\nu}. \qquad (29)$$

The above tensor operator satisfy the identity

$$O_{\mu\nu}\Delta^{\nu}_{\alpha} = g_{\mu\alpha}. \qquad (30)$$

Feynman propagator for Podolsky's electrodynamics

In the above identity, the Δ_{α}^{ν} is the inverse operator of $O_{\mu\nu}$. We must remember that

$$g_{\mu\nu} = \Theta_{\mu\nu} + \Omega_{\mu\nu}, \tag{31}$$

where the transverse and longitudinal projectors are, respectively, given by

$$\Theta_{\mu\nu} = g_{\mu\nu} - \Omega_{\mu\nu}, \tag{32}$$

$$\Omega_{\mu\nu} = \partial_{\mu}\partial_{\nu}/\Box. \tag{33}$$

Now we are able to propose the following forma for the inverse operator $\Delta_{\mu\nu}$ in terms os the known projectors, that is

$$\Delta_{\alpha}^{\nu} = a\Theta_{\alpha}^{\nu} + b\Omega_{\alpha}^{\nu}. \tag{34}$$

where *a* and *b* are unknown constants that must be determined.

Feynman propagator for Podolsky's electrodynamics

The transverse and longitudinal projectors satisfy the following closed algebra:

	Θ^{ν}_{α}	Ω^{ν}_{α}
$\Theta_{\mu\nu}$	$\Theta_{\mu\alpha}$	0
$\Omega_{\mu\nu}$	0	$\Omega_{\mu\alpha}$

Now, replacing the equation (34) in (30) and performing the tensor contractions, we find the Feynman propagator:

$$\langle A_{\nu} A_{\alpha} \rangle = -\frac{i}{p^2} \left[\frac{1}{\left(1 - \theta^2 p^2\right)} \Theta^{\nu}_{\alpha} - \xi \Omega^{\nu}_{\alpha} \right]. \tag{35}$$

Stability and causality analyses

- The propagator give us the following dispersion relations:

$$p_0^2 = \mathbf{p}^2, \tag{36}$$

$$p_0^2 = \mathbf{p}^2 + M_p^2, \tag{37}$$

where the second dispersion relation represents a massive mode, whose mass $M_p = 1/\theta$ is called Podolsky's mass.

- We have to analyze the dispersion relation (37), whose solutions are:

$$p_0 = \pm\sqrt{\mathbf{p}^2 + M_p^2}. \tag{38}$$

The positive energy $p_0 > 0$ ensure stability.

Stability and causality analyze

Group velocity and front velocity

$$\mathbf{u}_g = \frac{\partial p_0}{\partial \mathbf{p}} = \frac{|\mathbf{p}|}{\sqrt{\left(|\mathbf{p}|^2 + M_p^2\right)}} \leq 1, \tag{39}$$

$$u_{\text{front}} = \lim_{|\mathbf{p}|\to\infty} \frac{p_0}{|\mathbf{p}|} = \sqrt{1 + M_p^2 / |\mathbf{p}|^2} = 1. \tag{40}$$

For the causality, we have $u_g = |\mathbf{u}_g| \leq 1$ and $u_{frente} \leq 1$, so causality is preserved.

These results show us the the Podolsky's electrodynamics is stable and causal.

Analyzing the LV causality

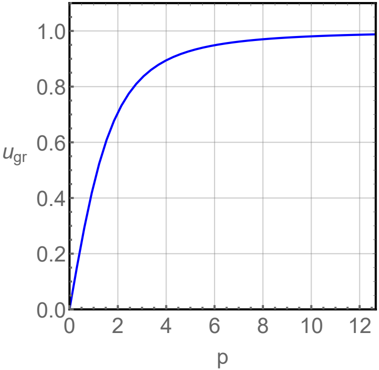


Figure: Group velocity

This dispersion relation does not describe a standard photon. It describes a causal Podolsky excitation. It is called "exotic" (different but not unphysical).

Unitarity analysis

The saturated propagator is given by

$$SP = J^{\nu}\,\mathrm{Re}\,s\left[i\Delta_{\nu\alpha}\right]J^{\alpha}, \tag{41}$$

where the current satisfies the conservation law: $\partial^{\nu}J_{\nu} = 0$. Now, consider the propagator

$$\langle A_{\nu}A_{\alpha}\rangle = -\frac{i}{p^2}\left[\frac{1}{\left(1-\theta^2 p^2\right)}\Theta^{\nu}_{\alpha} - \xi\Omega^{\nu}_{\alpha}\right]. \tag{42}$$

The saturation together with the currents is given by

$$SP = \mathrm{Re}\,s\left[-\frac{i}{p^2}\frac{1}{\left(1-\theta^2 p^2\right)}J^2\right]. \tag{43}$$

Unitarity analysis

For the first pole, $p^2 = 0$, which is the same as in Maxwell theory, the calculation of the residuo gives us the saturation:

$$SP_{p^2=0} = i\left(\mathbf{J}^2 - J_0^2\right). \qquad (44)$$

In the pole $p^2 = 0\rangle\ p_0^2 = \mathbf{p}^2$ and from the current conservation law, $\left(p_0 J_0 = \mathbf{p}\cdot\mathbf{J}\right)$,we obtain

$$SP_{(p^2=0)} = \frac{i}{\left|\mathbf{p}\right|^2}\left|\mathbf{p}\times\mathbf{J}\right|^2 > 0. \qquad (45)$$

This implies that the excitations associated with this propagating mode are unitary.

Unitarity analysis

For the second pole, $p^2 = 1/\theta^2$, the calculations lead to

$$SP_{(p^2=1/\theta^2)} = -i\left(\mathbf{J}^2 - J_0^2\right). \tag{46}$$

Using $p_0 J_0 = \mathbf{p}\cdot\mathbf{J}$ and $p_0^2 = 1/\theta^2 + \mathbf{p}^2$ the equation (46) may be rewrite as:

$$SP = -\frac{i}{\mathbf{p}^2 + 1/\theta^2}\left(\frac{\mathbf{J}^2}{\theta^2} + |\mathbf{p}\times\mathbf{J}|^2 + \right) < 0. \tag{47}$$

Thus, the saturated propagator is less than zero.
These states with negative norm (ghost states) are typical from higher derivative theories.

A generalized model involving anisotropic Podolsky and Lee-Wick terms

There are two other CPT-even dimension-six Lagrangian structures endowed with two additional derivatives, beyond the LV term $\partial_\sigma F^{\sigma\beta}\partial_\lambda F^{\lambda\alpha}D_{\beta\alpha}$, are they

$$F_{\mu\nu}\partial_\alpha\partial_\beta F^{\mu\nu}D^{\alpha\beta}\,, \quad \partial_\sigma F^{\sigma\lambda}\partial_\mu F_{\nu\lambda}D^{\mu\nu}\,. \tag{48}$$

In case the tensor $D_{\beta\alpha}$ is diagonal, it becomes proportional to the usual Lee-Wick term, that is,

$$F_{\mu\nu}\partial_\alpha\partial_\beta F^{\mu\nu}D^{\alpha\beta} = D_{00}(F_{\mu\nu}\Box F^{\mu\nu})\,. \tag{49}$$

In principle, the most general LV dimension-6 electrodynamics is

$$\begin{aligned}\mathcal{L} \;\; = \;\; & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\theta^2}{2}\partial_\alpha F^{\alpha\beta}\partial_\lambda F^{\lambda}{}_{\beta} + \eta_1^2 D_{\beta\alpha}\partial_\sigma F^{\sigma\beta}\partial_\lambda F^{\lambda\alpha} \\ & + \eta_2^2 D^{\beta\alpha}\partial_\sigma F^{\sigma\lambda}\partial_\beta F_{\alpha\lambda} + \frac{1}{2\xi}(\partial_\mu A^\mu)^2\,. \end{aligned} \tag{50}$$

Paper published

PHYSICAL REVIEW D **97**, 115043 (2018)

Maxwell electrodynamics modified by *CPT*-even and Lorentz-violating dimension-6 higher-derivative terms

Rodolfo Casana,[1,*] Manoel M. Ferreira, Jr.,[1,†] Letícia Lisboa-Santos,[1,‡] Frederico E. P. dos Santos,[2,§] and Marco Schreck[1,∥]

[1]*Departamento de Física, Universidade Federal do Maranhão (UFMA), Campus Universitário do Bacanga, São Luís, Maranhão 65080-805, Brazil*

[2]*Coordenação do Curso Interdisciplinar em Ciência e Tecnologia, Universidade Federal do Maranhão (UFMA), Campus Universitário do Bacanga, São Luís, Maranhão 65080-805, Brazil*

(Received 25 February 2018; revised manuscript received 18 May 2018; published 28 June 2018)

In this paper, we investigate an electrodynamics in which the physical modes are coupled to a Lorentz-violating background by means of a higher-derivative term. We analyze the modes associated with the dispersion relations obtained from the poles of the propagator. More specifically, we study Maxwell's electrodynamics modified by a Lorentz-violating operator of mass dimension 6. The modification has the form $D_{\beta\alpha}\partial_\sigma F^{\sigma\beta}\partial_\lambda F^{\lambda\alpha}$; i.e., it possesses two additional derivatives coupled to a *CPT*-even tensor $D_{\beta\alpha}$ that plays the role of the fixed background. We first evaluate the propagator and obtain the dispersion relations of the theory. By doing so, we analyze some configurations of the fixed background and search for sectors where the energy is well defined and causality is assured. A brief analysis of unitarity is included for particular configurations. Afterward, we perform the same kind of analysis for a more general dimension-6 model. We conclude that the modes of both Lagrange densities are possibly plagued by physical problems, including causality and unitarity violation, and that signal propagation may become physically meaningful only in the high-momentum regime.

DOI: 10.1103/PhysRevD.97.115043

Classification under CPT

$$\mathcal{L}_{LV} = \eta_1^2 \partial_\sigma F^{\sigma\beta} \partial_\lambda F^{\lambda\alpha} \ D_{\beta\alpha}, \tag{51}$$

- Under $C-$ transformation: $E \to -E$ and $B \to -B$.
- Under $P-$ transformation: $E \to -E$ and $B \to +B$.
- Under $T-$ transformation: $E \to E$ and $B \to -B$

- Coefficient D_{00} :

$$\eta^2 \partial_\sigma F^{\sigma 0} \partial_\lambda F^{\lambda 0} \ D_{00}, \tag{52}$$

sum of Einstein

$$\left(\eta^2 \partial_0 F^{00} \partial_0 F^{00} + \eta^2 \partial_0 F^{00} \partial_i F^{i0} + \eta^2 \partial_i F^{i0} \partial_0 F^{00} + \eta^2 \partial_i F^{i0} \partial_j F^{j0}\right) \ D_{00},$$

as $F^{00} = 0$, $F^{i0} = E^i$ we have:

$$\boxed{\eta^2 \partial_i E^i \partial_j E^j D_{00}.} \tag{53}$$

Classification under CPT

- $C-$ Transformation.

$$C\eta^2\partial_i E^i\partial_j E^j \; D_{00} C^{-1},$$
$$\eta^2\partial_i\left(-E^i\right)\partial_j\left(-E^j\right) \; D_{00},$$
$$\eta^2\partial_i E^i\partial_j E^j \; D_{00}, \tag{54}$$

it is invariant under transformation C.

- $P-$ Transformation.

$$P\eta^2\partial_i E^i\partial_j E^j \; D_{00} P^{-1}.$$
$$\eta^2\left(-\partial_i\right)\left(-E^i\right)\left(-\partial_j\right)\left(-E^j\right) \; D_{00}.$$
$$\eta^2\partial_i E^i\partial_j E^j \; D_{00}, \tag{55}$$

it is invariant under transformation P.

Classification under CPT

- $T-$ Transformation.

$$\eta^2\left(\partial_i\right)\left(E^i\right)\left(\partial_j\right)\left(E^j\right)\ D_{00},$$
$$\eta^2\partial_i E^i\partial_j E^j\ D_{00}, \tag{56}$$

it is invariant under transformation T. we have the coefficient D_{00} is invariant under the transformations CPT, so it is $CPT-$even.

- For D_{i0} :

$$\eta^2\partial_\sigma F^{\sigma i}\partial_\lambda F^{\lambda 0}\ D_{i0}, \tag{57}$$

sum of Einstein

$$\left(\eta^2\partial_0 F^{0i}\partial_0 F^{00} + \eta^2\partial_0 F^{0i}\partial_j F^{j0} + \eta^2\partial_j F^{ji}\partial_0 F^{00} + \eta^2\partial_j F^{ji}\partial_k F^{k0}\right)\ D_{i0},$$

as $F^{00}=0$, $F^{i0}=E^i$, $F^{ji}=\varepsilon_{jil}B^l$ we have:

$$-\eta^2\partial_0 E^i\partial_j E^j\ D_{i0} + \eta^2\varepsilon_{jil}\partial_j B^l\partial_k E^k\ D_{i0}. \tag{58}$$

Classification under CPT

- $C-$ Transformation.

$$-\eta^2\partial_0\left(-E^i\right)\partial_j\left(-E^j\ \right)D_{i0}+\eta^2\varepsilon_{jil}\partial_j\left(-B^l\right)\partial_k\left(-E^k\ \right)D_{i0},$$

$$-\eta^2\partial_0 E^i\partial_j E^j\ D_{i0}+\eta^2\varepsilon_{jil}\partial_j B^l\partial_k E^k\ D_{i0}, \tag{59}$$

- $P-$ Transformation.

$$-\eta^2\left[\partial_0\left(-E^i\right)\left(-\partial_j\right)\left(-E^j\right)-\varepsilon_{jil}\left(-\partial_j\right)\left(B^l\right)\left(-\partial_k\right)\left(-E^k\right)\right]D_{i0},$$

$$\eta^2\partial_0 E^i\partial_j E^j\ D_{i0}-\eta^2\varepsilon_{jil}\partial_j B^l\partial_k E^k D_{i0}, \tag{60}$$

- $T-$ Transformation.

$$-\eta^2\left(-\partial_0\right)\left(E^i\right)\left(\partial_j\right)\left(E^j\right)\ D_{i0}+\eta^2\varepsilon_{jil}\left(\partial_j\right)\left(-B^l\right)\left(\partial_k\right)\left(E^k\right)\ D_{i0},$$

$$\eta^2\partial_0 E^i\partial_j E^j D_{i0}-\eta^2\varepsilon_{jil}\partial_j B^l\partial_k E^k\ D_{i0}, \tag{61}$$

We have the coefficient D_{i0} is invariant under the transformations CPT, so it is $CPT-$even.

Classification under CPT

- Coefficient D_{0i} :

$$\eta^2\partial_\sigma F^{\sigma 0}\partial_\lambda F^{\lambda i} D_{0i},$$

$$-\eta^2\partial_j E^j\partial_0 E^i \ D_{0i} + \eta^2\partial_j E^j \varepsilon_{kil}\partial_k B^l \ D_{0i}. \tag{62}$$

- $C-$ Transformation.

$$-\eta^2\partial_j E^j\partial_0 E^i \ D_{0i} + \eta^2\partial_j E^j \varepsilon_{kil}\partial_k B^l \ D_{0i}, \tag{63}$$

- $P-$ Transformation.

$$\eta^2\partial_j E^j\partial_0 E^i \ D_{0i} - \eta^2\partial_j E^j \varepsilon_{kil}\partial_k \left(B^l\right) \ D_{0i}, \tag{64}$$

- $T-$ Transformation.

$$\eta^2\partial_j E^j\partial_0 E^i D_{0i} - \eta^2\partial_j E^j \varepsilon_{kil}\partial_k B^l D_{0i}, \tag{65}$$

We have the coefficient D_{0i} under the transformations CPT is $CPT-$even.

Classification under CPT

- Coefficient D_{ij} :

$$\eta^2\partial_\sigma F^{\sigma i}\partial_\lambda F^{\lambda j}\ D_{ij},$$

$$\eta^2\left(\partial_0 E^i\partial_0 E^j-\varepsilon_{kjn}\partial_0 E^i\partial_k B^n-\varepsilon_{kin}\partial_k B^n\partial_0 E^j+\varepsilon_{kin}\varepsilon_{ljh}\partial_k B^n\partial_l B^h\right)D_{ij},$$

- $C-$ Transformation.

$$\eta^2\left(\partial_0 E^i\partial_0 E^j-\varepsilon_{kjn}\partial_0 E^i\partial_k B^n-\varepsilon_{kin}\partial_k B^n\partial_0 E^j+\varepsilon_{kin}\varepsilon_{ljh}\partial_k B^n\partial_l B^h\right)D_{ij}, \tag{66}$$

- $P-$ Transformation.

$$\eta^2\left(\partial_0 E^i\partial_0 E^j-\varepsilon_{kjn}\partial_0 E^i\partial_k B^n-\varepsilon_{kin}\partial_k B^n\partial_0 E^j+\varepsilon_{kin}\varepsilon_{ljh}\partial_k B^n\partial_l B^h\right)D_{ij}, \tag{67}$$

- $T-$ Transformation.

$$\eta^2\left(\partial_0 E^i\partial_0 E^j-\varepsilon_{kjn}\partial_0 E^i\partial_k B^n-\varepsilon_{kin}\partial_k B^n\partial_0 E^j+\varepsilon_{kin}\varepsilon_{ljh}\partial_k B^n\partial_l B^h\right)D_{ij}. \tag{68}$$

Classification under CPT

- Results:

	C	P	T	CPT
D_{00}	$+$	$+$	$+$	$+$
D_{i0}	$+$	$-$	$-$	$+$
D_{0i}	$+$	$-$	$-$	$+$
D_{ij}	$+$	$+$	$+$	$+$

So, this model is CPT-even.

The behavior for the group velocity modulus

- **B** and **C** parallel

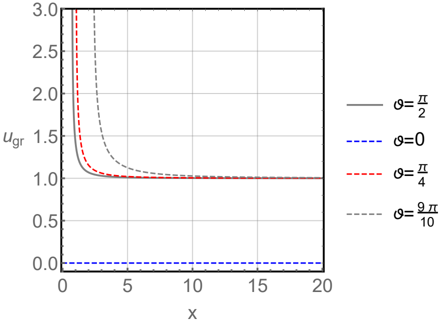


Figure: Group velocity for $\alpha > 0$

This figure represents SPURIOUS dispersion relations, since there is causality violation.

The behavior for the group velocity modulus

- **B** and **C** antiparallel does not exhibit any singularity, is exotic, with no causality violation. Podolsky-like DR.

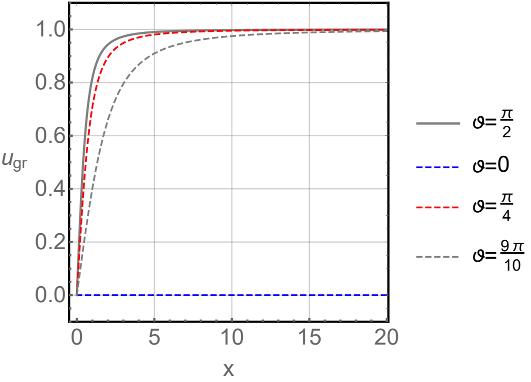


Figure: Group velocity for $\alpha < 0$

The behavior for the group velocity modulus

This figure shows SPURIOUS and EXOTIC excitations, for different parameter values.

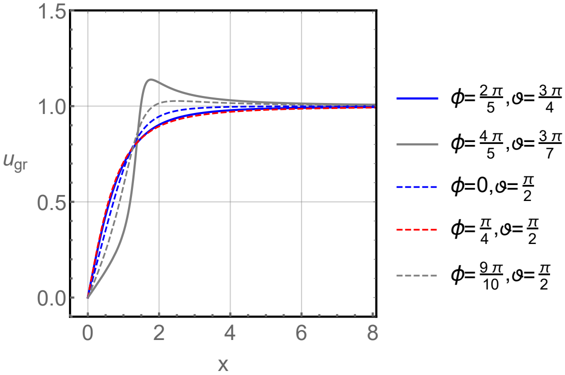


Figure: group velocity for different angles of the positive mode

The behavior for the group velocity modulus

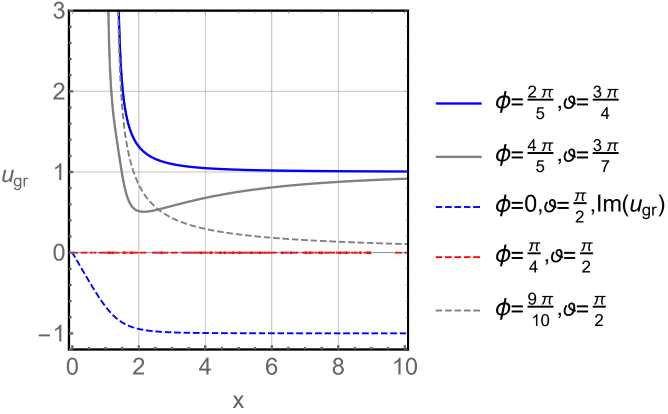


Figure: group velocity for different angles of the negative mode

This figure shows SPURIOUS excitations, for different parameter values.

Conclusions for parity-even anisotropic sector

- The positive mode group velocity has a maximum and approaches one and becomes greater than the first breaking causality for a certain range of parameters.
- For the same angular values, the negative mode group velocity decreases from its initial singularity to a minimum and finally approaches 1 from below.
- For other values of the angles, it does not reveal any maximum or minimum, with the group velocity from the negative mode approaching 1 from above and the positive mode approaching 1 from below. For this choice of parameters, the positive mode is exotic.
- It has also been found that the negative mode does not propagate to certain angles. For example, the group velocity disappears identically for $\vartheta = \pi/2$ and $\varphi = \pi/4$, and for $\vartheta = \pi/2$ and $\varphi < [\pi + arcsin(1/x^2)]/2$, it leads to complex values.

Maxwell electrodynamics modified by a CPT-odd dimension-5 higher-derivative term

- Therefore, in the present work, we study Maxwell electrodynamics modified by a CPT-odd, dimension-5 nonminimal SME term represented by the Carroll-Field-Jackiw-like (CFJ-like) term of the Lagrange density:

$$\frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}A_{\lambda}(\hat{k}_{AF})_{\kappa}F_{\mu\nu}\,. \tag{69}$$

As a first investigation, we consider the special case for the LV background

$$(\hat{k}_{AF})_{\kappa}=\tilde{D}_{\kappa}\Box\,, \tag{70a}$$

and the new Lagrange density has the form

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}D_{\kappa}A_{\lambda}\Box F_{\mu\nu}+\frac{1}{2\xi}(\partial_{\mu}A^{\mu})^{2}\,. \tag{71}$$

Feynman propagator for Maxwell electrodynamics modified

By performing suitable partial integrations and neglecting boundary terms, the latter can be written as

$$\mathcal{L} = \frac{1}{2} A^{\mu} O_{\mu\nu} A^{\nu} \,, \tag{72a}$$

with the differential operator

$$O_{\mu\nu} = \Box \left(\Theta_{\mu\nu} - 2L_{\mu\nu} - \frac{1}{\xi} \Omega_{\mu\nu} \right) \,, \tag{72b}$$

sandwiched in between two vector fields. Here we introduced the symmetric transversal and longitudinal projectors, $\Theta_{\mu\nu}$ and $\Omega_{\mu\nu}$, respectively:

$$\Theta_{\mu\nu} \equiv \eta_{\mu\nu} - \Omega_{\mu\nu} \,, \quad \Omega_{\mu\nu} \equiv \frac{\partial_{\mu}\partial_{\nu}}{\Box} \,, \tag{73}$$

while the Lorentz-violating part is described by the antisymmetric and dimensionless operator,

$$L_{\mu\nu} \equiv \epsilon_{\mu\nu\kappa\lambda} D^{\kappa} \partial^{\lambda} \,. \tag{74}$$

Feynman propagator for Maxwell electrodynamics modified

- Now we intend to evaluate the propagator of the theory, i.e., we should find the Green's function $\Delta_{\alpha\beta}$, which is the inverse of the differential operator $O_{\mu\nu}$, from the condition

$$O_{\mu\sigma}\Delta^{\sigma}{}_{\nu} = \eta_{\mu\nu}\,. \tag{75}$$

We propose the following *Ansatz*:

$$\Delta^{\sigma}{}_{\nu} = a\Theta^{\sigma}{}_{\nu} + bL^{\sigma}{}_{\nu} + c\Omega^{\sigma}{}_{\nu} + dD^{\sigma}D_{\nu} + e\left(D^{\sigma}\partial_{\nu} + D_{\nu}\partial^{\sigma}\right), \tag{76}$$

where the parameters $a \ldots e$ are expected to be scalar operators.

Feynman propagator for Maxwell electrodynamics modified

- The Feynman propagator is defined by the vacuum expectation value of the time-ordered product of field operators evaluated at distinct spacetime points,

$$\mathrm{i}(D_F)_{\alpha\beta}(x-y) \equiv \langle 0|\, T\left(A_\alpha(x)A_\beta(y)\right)|0\rangle\,. \tag{77}$$

The form of the propagator in momentum:

$$\Delta_{\mu\sigma}(p) = \frac{-\mathrm{i}}{p^2(1+4\Upsilon(p))}\left\{\eta_{\mu\sigma} - 2\mathrm{i}\epsilon_{\mu\sigma\kappa\lambda}D^\kappa p^\lambda - \left[1-4(D\cdot p)^2\right]\frac{p_\mu p_\sigma}{p^2}\right.$$

$$\left. -\xi\left(1+4\Upsilon(p)\right)\frac{p_\mu p_\sigma}{p^2} + 4p^2 D_\mu D_\sigma - 4(D\cdot p)\left[D_\mu p_\sigma + D_\sigma p_\mu\right]\right\}, \tag{78}$$

with

$$\Upsilon(p) = D^2p^2 - (D\cdot p)^2\,. \tag{79}$$

Dispersion relations

The poles of the propagator provide two dispersion equations for this model,

$$p^2 = 0\,, \qquad (80a)$$

$$1 + 4\left[D^2 p^2 - (D\cdot p)^2\right] = 0\,, \qquad (80b)$$

as usual in theories with higher-dimensional operators.

- The second equation contains information on the higher-derivative dimension-5 term. It is reasonable to compare it to the dispersion equation obtained for the dimension-4 MCFJ theory , given in terms of the CFJ background vector $(k_{AF})^\mu$:

$$p^4 + p^2 k_{AF}^2 - (k_{AF}\cdot p)^2 = 0\,. \qquad (81)$$

Analyze of some sectors of the theory

- For a purely timelike background, $D_\gamma = (D_0, 0)_\gamma$, we have

$$\mathbf{p}^2 = \frac{D_0^2}{4}\,, \tag{82}$$

  which does not correspond to a propagating mode. It is a nonphysical DR, as it does not represent a relation between energy and momentum.

- This property is an important difference between the dimension-5 model under consideration and MCFJ theory. The latter exhibits a DR associated with a timelike background vector.

Spacelike dispersion relations

- For a purely spacelike background, $D_\gamma = (0, \mathbf{D})_\gamma$, the corresponding DR is

$$p_0 = \frac{1}{|\mathbf{D}|}\sqrt{\frac{1}{4} + |\mathbf{D}\times\mathbf{p}|^2}\,, \tag{83}$$

which can also be written as

$$p_0 = \frac{1}{|\mathbf{D}|}\sqrt{\frac{1}{4} + \mathbf{D}^2\mathbf{p}^2\sin^2\alpha}\,, \tag{84}$$

with the angle α enclosed by $\mathbf{D}$ and $\mathbf{p}$:

$$\mathbf{D}\cdot\mathbf{p} = |\mathbf{D}||\mathbf{p}|\cos\alpha\,. \tag{85}$$

This is a DR that is compatible with the propagation of signals, whose properties need to be examined.

Causality analysis

The classical causality is characterized by the behavior of the group and front velocity $\mathbf{u}_{\mathrm{gr}}$ and u_{fr}, respectively, where

$$\mathbf{u}_{\mathrm{gr}} \equiv \frac{\partial p_0}{\partial \mathbf{p}}\,, \quad u_{\mathrm{fr}} \equiv \lim_{|\mathbf{p}|\mapsto\infty} \frac{p_0}{|\mathbf{p}|}\,. \tag{86}$$

Classical causality is established as long as both $u_{\mathrm{gr}} \equiv |\mathbf{u}_{\mathrm{gr}}| \leq 1$ and $u_{\mathrm{fr}} \leq 1$. We now evaluate these characteristic velocities for DR (83). The front velocity is

$$u_{\mathrm{fr}} = \lim_{|\mathbf{p}|\mapsto\infty} \sqrt{\frac{1}{4\mathbf{D}^2\mathbf{p}^2} + \sin^2\alpha} = \sin\alpha\,, \tag{87}$$

as $\alpha \in [0, \pi]$.

Causality analysis

Group velocity whose magnitude is

$$u_{\mathrm{gr}} = \frac{\sin\alpha}{\sqrt{1/(4x^2) + \sin^2\alpha}}\,, \tag{88}$$

where $x \equiv |\mathbf{D}||\mathbf{p}|$ is a dimensionless parameter. Large momenta correspond to large x. Hence,

$$\lim_{|\mathbf{p}|\mapsto\infty} u_{\mathrm{gr}} = \lim_{x\mapsto\infty} u_{\mathrm{gr}} = 1\,, \tag{89}$$

independently of the angle α.

- As $u_{\mathrm{gr}} \leq 1$ and $u_{\mathrm{fr}} \leq 1$, classical causality is established.
- If a DR does not approach the limit of standard electromagnetic waves for zero Lorentz violation, it will be called "exotic."
- The latter are not necessarily unphysical, but they decouple in the low-energy regime.

The behavior of the magnitude of the group velocity

The graph shows a monotonically increasing group velocity that reaches the asymptotic value 1, which is a behavior in accordance with causality.

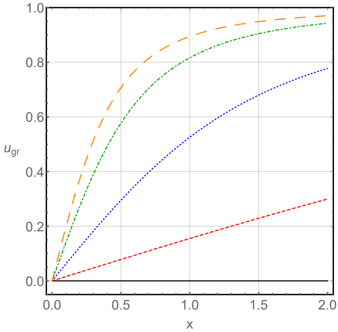


Figure: Magnitude of the group velocity of Eq. (88) for the spacelike case with $\alpha = 0$ (black, plain), $\alpha = \pi/40$ (red, dashed), $\alpha = \pi/10$ (blue, dotted), $\alpha = \pi/4$ (green, dashed-dotted), and $\alpha = \pi/2$ (orange, long dashes).

Unitarity analysis

- The unitarity at tree-level, which is performed by means of the saturated propagator SP .

$$SP \equiv J^{\mu}\mathrm{i}\Delta_{\mu\nu}J^{\nu} \,. \tag{90}$$

- The current J^{μ} is assumed to be real and satisfies the conservation law $\partial_{\mu}J^{\mu} = 0$, which in momentum space reads $p_{\mu}J^{\mu} = 0$.
- The saturation is

$$SP = -\mathrm{i}\left\{\frac{J^2 + 4p^2(J \cdot D)^2}{p^2\left[1 + 4(D^2p^2 - (D \cdot p)^2)\right]}\right\}\,, \tag{91}$$

where

$$J^{\mu}L_{\mu\alpha}J^{\alpha} = -\mathrm{i}J^{\mu}\epsilon_{\mu\alpha\kappa\lambda}J^{\alpha}D^{\kappa}p^{\lambda} = 0. \tag{92}$$

Hence, the only Lorentz-violating contributions of the propagator that have an impact on unitarity are the denominators and the symmetric term, $D_{\mu}D_{\nu}$.

Unitarity for the spacelike configuration

- For the configuration, $D_\mu = (0, \mathbf{D})_\mu$, where

$$p^2 = \frac{1 - 4(\mathbf{D}\cdot\mathbf{p})^2}{4\mathbf{D}^2}\,, \tag{93}$$

we obtain the residue at this pole

$$\mathrm{Res}(SP)\big|_{p^2=\frac{1-4(\mathbf{D}\cdot\mathbf{p})^2}{4\mathbf{D}^2}} =$$
$$= \mathrm{i}\left\{\frac{\left(1-4(\mathbf{D}\cdot\mathbf{p})^2\right)\left[4(\mathbf{D}\cdot\mathbf{J})^2(\mathbf{D}\times\mathbf{p})^2-(\mathbf{D}\times\mathbf{J})^2\right]-4\mathbf{D}^4(\mathbf{J}\times\mathbf{p})^2}{\mathbf{D}^2\left(1-4(\mathbf{D}\cdot\mathbf{p})^2\right)\left[1+4(\mathbf{D}\times\mathbf{p})^2\right]}\right\} \tag{94}$$

There are configurations for which the imaginary part of the latter is positive.

Spacelike configuration

- p parallel to J

We have

$$\mathrm{Res}(SP)\big|_{p^2=\frac{1-4(\mathbf{D}\cdot\mathbf{p})^2}{4\mathbf{D}^2},\mathbf{p}\parallel\mathbf{J}} = \mathrm{i}\left[-\frac{\mathbf{J}^2}{1+4(\mathbf{D}\times\mathbf{p})^2}+\frac{(\mathbf{D}\cdot\mathbf{J})^2}{\mathbf{D}^2}\right]. \tag{95}$$

The first term can be suppressed for large momenta as long as $\mathbf{D}\nparallel\mathbf{p}$.

- As the second contribution does not depend on the momentum, the imaginary part of the residue can be positive for large enough momenta.
- Hence, there are configurations of background field, large momenta, and external current for which unitarity is valid.

Spacelike configuration

- θ is the angle between D and J

We have $(\mathbf{D}\cdot\mathbf{J})^2 = \mathbf{D}^2\mathbf{J}^2\cos^2\theta$ and $(\mathbf{D}\times\mathbf{p})^2 = \mathbf{D}^2\mathbf{p}^2\sin^2\theta$. Thus, the residue reads

$$\mathrm{Res}(SP)\big|_{p^2=\frac{1-4(\mathbf{D}\cdot\mathbf{p})^2}{4\mathbf{D}^2},\mathbf{p}\|\mathbf{J}} = \mathrm{i}\mathbf{J}^2\sin^2\theta\left(\frac{4(\mathbf{D}\cdot\mathbf{p})^2-1}{4(\mathbf{D}\times\mathbf{p})^2+1}\right), \tag{96}$$

whose imaginary part is positive for

$$(\mathbf{D}\cdot\mathbf{p})^2 > \frac{1}{4}\,. \tag{97}$$

The latter condition assures unitarity.

Maxwell electrodynamics modified by a CPT-odd dimension-5 higher derivative term: a second model

As a second possibility, we propose the more sophisticated choice

$$(\hat{k}_{AF})_\kappa = (k_{AF}^{(5)})_\kappa{}^{\alpha_1\alpha_2}\partial_{\alpha_1}\partial_{\alpha_2} = T_\kappa T^{\alpha_1} T^{\alpha_2}\partial_{\alpha_1}\partial_{\alpha_2} = T_\kappa (T\cdot\partial)^2 \,, \tag{98}$$

where T_κ is a Lorentz-violating four-vector whose mass dimension is

$$[T_\kappa^3] = -1. \tag{99}$$

Modifying Maxwell's theory by including this term into its Lagrange density, leads to a higher-derivative (dimension-5) anisotropic MCFJ-like theory described by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}A_\lambda T_\kappa (T\cdot\partial)^2 F_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \,, \tag{100}$$

it is directly linked to the photon sector of Myers-Pospelov theory .

Paper published

PHYSICAL REVIEW D **100,** 055036 (2019)

Maxwell electrodynamics modified by a *CPT*-odd dimension-five higher-derivative term

Manoel M. Ferreira, Jr.,[*] Letícia Lisboa-Santos,[†] Roberto V. Maluf,[‡] and Marco Schreck[§]

Departamento de Física, Universidade Federal do Maranhão, Campus Universitário do Bacanga, São Luís, Maranhão, 65080-805, Brazil and Departamento de Física, Universidade Federal do Ceará, Campus Universitário do Piauí, Caixa Postal 6030, Fortaleza, Ceará, 60455-760, Brazil

(Received 1 April 2019; published 26 September 2019)

In this paper, we consider an electrodynamics of higher derivatives coupled to a Lorentz-violating background tensor. Specifically, we are interested in a dimension-five term of the *CPT*-odd sector of the nonminimal Standard Model extension. By a particular choice of the operator $\hat{k}_{AF}$, we obtain a higher-derivative version of the Carroll-Field-Jackiw (CFJ) term, $\frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}A_{\lambda}D_{\kappa}\Box F_{\mu\nu}$, with a Lorentz-violating background vector D_{κ}. This modification is subject to being investigated. We calculate the propagator of the theory and from its poles, we analyze the dispersion relations of the isotropic and anisotropic sectors. We verify that classical causality is valid for all parameter choices, but that unitarity of the theory is generally not assured. The latter is found to break down for certain configurations of the background field and momentum. In an analog way, we also study a dimension-five anisotropic higher-derivative CFJ term, which is written as $\epsilon^{\kappa\lambda\mu\nu}A_{\lambda}T_{\kappa}(T\cdot\partial)^2F_{\mu\nu}$ and is directly linked to the photon sector of Myers-Pospelov theory. Within the second model, purely timelike and spacelike T_{κ} are considered. For the timelike choice, one mode is causal, whereas the other is noncausal. Unitarity is conserved, in general, as long as the energy stays real—even for the noncausal mode. For the spacelike scenario, causality is violated when the propagation direction lies within certain regimes. However, there are particular configurations preserving unitarity and strong numerical indications exist that unitarity is guaranteed for all purely spacelike configurations. The results improve our understanding of nonminimal *CPT*-odd extensions of the

Conclusions

- The analysis of the dispersion relations obtained from the propagator poles revealed that signals do not propagate at all for the purely timelike sector.
- The modes of the purely spacelike sector decouple from the theory for low energies and only propagate in the high-energy regime.
- For this sector, classical causality is preserved for any choice of background coefficients.
- It was found that unitarity can be preserved in some special cases. In general, however, the dispersion relations describe nonunitary modes.
- This propagator possesses some analogy with that of MCFJ theory. But the dispersion relations and the related physics are different between these two models.
- The second dimension-five term examined was an anisotropic higher-derivative CFJ-like contribution that can be identified with the photon sector of Myers-Pospelov theory.

Bibliografia

- D. Colladay and V. A. Kostelecký,
  D. Colladay and V.A. Kostelecký, Phys. Rev. D **55**, 6760 (1997);
  D. Colladay and V.A. Kostelecký, Phys. Rev. D **58**, 116002 (1998);
  S.R. Coleman and S.L. Glashow, Phys. Rev. D **59**, 116008 (1999).
- R. Casana, M.M. Ferreira, Jr., L. Lisboa-Santos, F.E.P. dos Santos, and M. Schreck, Phys. Rev. D **97**, 115043 (2018).
- V.A. Kostelecký and M. Mewes, Phys. Rev. D **88**, 096006 (2013);
  M. Schreck, Phys. Rev. D **90**, 085025 (2014).
- C.M. Reyes, L.F. Urrutia, and J.D. Vergara, Phys. Lett. B **675**, 336 (2009).
- C.M. Reyes, Phys. Rev. D **87**, 125028 (2013).

Acknowledgment

CAPES

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

GFTPC UFMA
Grupo de Física Teórica de Partículas e Campos
Fazendo Ciência de Primeira Linha no Maranhão

UFMA
Universidade Federal do Maranhão
50 anos
1966 • 2016

FAPEMA