Nonminimal SME: an overview

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VIII Ciclo de Seminários do Curso de Física

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Outline

Introduction

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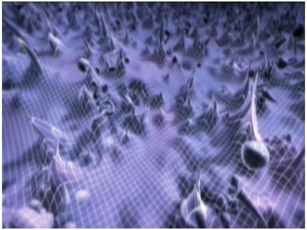
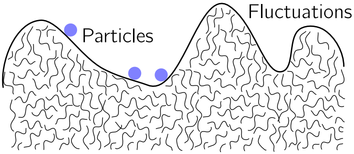
Motivation for Lorentz Violation

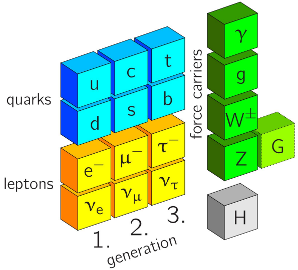
- The nature of spacetime.
  [Bernadotte, Klinkhamer (2007), Phys. Rev. D **75**, 024028]
- Lorentz violation from string theory.
  [Kostelecký, Samuel (1989), Phys. Rev. D **39**, 683]
- Loop quantum gravity theories.
  [R. Gambini and J. Pullin (1999) Phys. Rev. D **59**, 124021]
- Noncommutative spacetimes.
  $[x^{\mu}, x^{\nu}] = i\theta^{\mu\nu}$
  - [Carroll, Harvey, Kostelecký, Lane, Okamoto (2001), Phys. Rev. Lett. **87**, 141601]
  - [Schreck (2014), Journal of Physics: Conference Series 563, 012026]

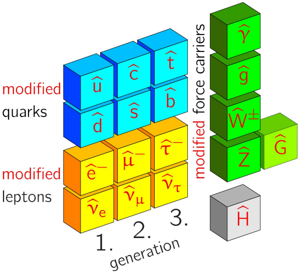




Outline

Properties of the SME





- Sub-Planckian effective description of Lorentz violation.
- Extension of Standard Model and General Relativity.

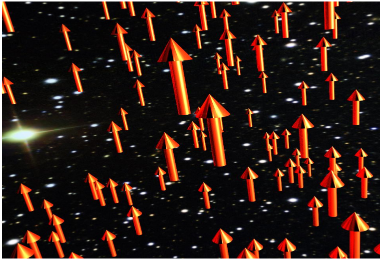
  [Colladay, Kostelecký (1998), Phys. Rev. D **58**, 116002; Kostelecký (2004), Phys. Rev. D **69**, 105009]

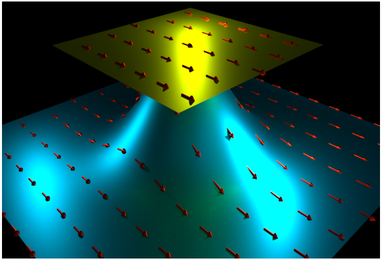
$$\mathcal{L}_{\text{SME}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{GR}} + \delta\mathcal{L}_{\text{LV}},$$

- The controlling coefficients:
  - Minimal $\mapsto$ contains all renormalizable operators ($d = 3, 4$)
  - Nonminimal $\mapsto$ contains all nonrenormalizable operators ($d > 4$)

Properties of the SME

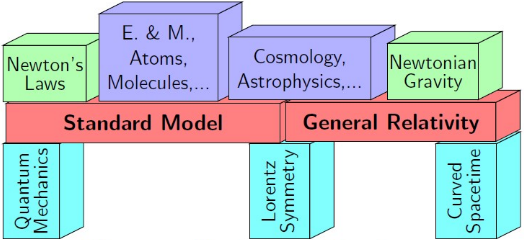
- CPT-violating contributions taken into account.
  [Greenberg (2002), hep-ph/0201258]
- Lorentz violation becomes manifest as background field.
- Background couples to fields and modifies them.
- Background: Nongravitational SME $\neq$ Gravitational SME.



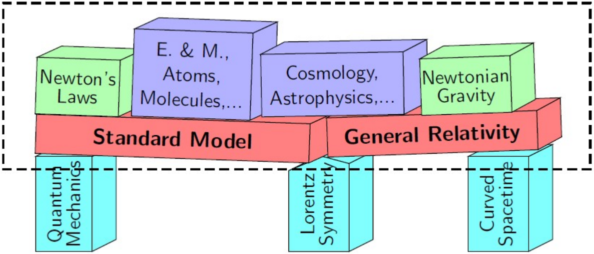


[R. Bluhm, N.L. Gagne (2008), hep-ph/0802.4071; R.V. Maluf, C.A.S. Almeida, R. Casana, and M.M. Ferreira Jr (2014), hep-th/1402.3554; R. Bluhm (2015), gr-qc/1401.4515]

Properties of the SME



SME = general description of LV at low energies



[M. Mewes, Fourth SME Summer School 2021]

Symmetries and Sectors of the SME

- The gauge symmetry $SU_c(3) \times SU_L(2) \times U_Y(1)$ should be respected

$$\underbrace{\overline{\overline{SU_c(3)}}}_{\substack{\downarrow \\ 8\ G^\alpha_\mu \\ \alpha = 1,\dots,8}} \times \underbrace{\overline{\overline{SU_L(2)}}}_{\substack{\downarrow \\ 3\ W^a_\mu \\ a = 1,\dots,3}} \times \underbrace{\overline{\overline{U_Y(1)}}}_{\substack{\downarrow \\ B_\mu}}$$

$$\text{Field strength tensors} = \begin{cases} G^\alpha_{\mu\nu} = \partial_\mu G^\alpha_\nu - \partial_\nu G^\alpha_\mu + g_3 f^\alpha_{\beta\gamma} G^\beta_\mu G^\gamma_\nu , \\ W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon_{abc} W^b_\mu W^c_\nu , \\ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu , \end{cases}$$

- The SM Lagrange density shown previously reads,

$$\mathcal{L}^{\mathrm{SM}} = \mathcal{L}^{\mathrm{SM}}_{\mathrm{Lepton}} + \mathcal{L}^{\mathrm{SM}}_{\mathrm{Quark}} + \mathcal{L}^{\mathrm{SM}}_{\mathrm{Yukawa}} + \mathcal{L}^{\mathrm{SM}}_{\mathrm{Higgs}} + \mathcal{L}^{\mathrm{SM}}_{\mathrm{Gauge}},$$

- And for the deviation from the exact Lorentz symmetry we have,

$$\begin{aligned} \delta\mathcal{L}_{\mathrm{LV}} \quad = \quad & \mathcal{L}^{\mathrm{CPT-par}}_{\mathrm{Lepton}} + \mathcal{L}^{\mathrm{CPT-impar}}_{\mathrm{Lepton}} + \mathcal{L}^{\mathrm{CPT-par}}_{\mathrm{Quark}} + \mathcal{L}^{\mathrm{CPT-impar}}_{\mathrm{Quark}} + \mathcal{L}^{\mathrm{CPT-par}}_{\mathrm{Yukawa}} \\ & + \mathcal{L}^{\mathrm{CPT-par}}_{\mathrm{Higgs}} + \mathcal{L}^{\mathrm{CPT-impar}}_{\mathrm{Higgs}} + \mathcal{L}^{\mathrm{CPT-par}}_{\mathrm{Gauge}} + \mathcal{L}^{\mathrm{CPT-impar}}_{\mathrm{Gauge}} . \end{aligned}$$

Exemples of Background Fields Couplings in SME

$$\mathcal{L}^{\text{CPT}-\text{par}}_{\text{Lepton}} = \frac{\mathrm{i}}{2}\left(c_L\right)_{\mu\nu AB}\,\bar{L}_A\gamma^\mu \overleftrightarrow{D^\nu} L_B + \frac{\mathrm{i}}{2}\left(c_R\right)_{\mu\nu AB}\,\bar{R}_A\gamma^\mu \overleftrightarrow{D^\nu} R_B\,,$$

$$\mathcal{L}^{\text{CPT}-\text{impar}}_{\text{Lepton}} = -\left(a_L\right)_{\mu AB}\,\bar{L}_A\gamma^\mu L_B - \left(a_L\right)_{\mu AB}\,\bar{R}_A\gamma^\mu R_B\,,$$

$$\begin{aligned}\mathcal{L}^{\text{CPT}-\text{par}}_{\text{Gauge}} &= -\frac{1}{2}\left(k_G\right)_{\kappa\lambda\mu\nu}\,\mathrm{tr}\left(G^{\kappa\lambda}G^{\mu\nu}\right) - \frac{1}{2}\left(k_W\right)_{\kappa\lambda\mu\nu}\,\mathrm{tr}\left(W^{\kappa\lambda}W^{\mu\nu}\right)\\ &\quad - \frac{1}{4}\left(k_B\right)_{\kappa\lambda\mu\nu}\,B^{\kappa\lambda}B^{\mu\nu},\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{\text{CPT}-\text{impar}}_{\text{Gauge}} &= \frac{1}{2}\left(k_3\right)_\kappa\,\epsilon^{\kappa\lambda\mu\nu}\mathrm{tr}\left(G_\lambda G_{\mu\nu} + \frac{2}{3}\mathrm{i}g_3 G_\lambda G_\mu G_\nu\right)\\ &\quad + \frac{1}{2}\left(k_2\right)_\kappa\,\epsilon^{\kappa\lambda\mu\nu}\mathrm{tr}\left(W_\lambda W_{\mu\nu} + \frac{2}{3}\mathrm{i}g_2 W_\lambda W_\mu W_\nu\right)\\ &\quad + \frac{1}{4}\left(k_1\right)_\kappa\,\epsilon^{\kappa\lambda\mu\nu}B_\lambda B_{\mu\nu} + \left(k_0\right)_\kappa\,B^\kappa.\end{aligned}$$

Single-Fermion Sector

- The Lagrange density for the single-fermion sector is of the form:

$$\mathcal{L} = \frac{1}{2}\bar{\psi}\left(\gamma^{\nu}\mathrm{i}\partial_{\nu} - m_{\psi} + \hat{\mathcal{Q}}\right)\psi + \mathrm{H.c.},$$

$$\hat{\mathcal{Q}} = \hat{\mathcal{S}} + \mathrm{i}\hat{\mathcal{P}}\gamma_5 + \hat{\mathcal{V}}^{\mu}\gamma_{\mu} + \hat{\mathcal{A}}^{\mu}\gamma_5\gamma_{\mu} + \frac{1}{2}\hat{\mathcal{T}}^{\mu\nu}\sigma_{\mu\nu}.$$

- For instance, we can decompose the nonminimal $\hat{\mathcal{V}}^{\mu}$ and $\hat{\mathcal{A}}^{\mu}$ operators:

$$\hat{\mathcal{V}}^{\mu} = \underbrace{\mathcal{V}^{(3)\mu} + \mathcal{V}^{(4)\mu\nu}(\mathrm{i}\partial_{\nu})}_{\text{Minimal}} + \underbrace{\mathcal{V}^{(5)\mu\nu\alpha}(\mathrm{i}\partial_{\nu})(\mathrm{i}\partial_{\alpha}) + \ldots}_{\text{Nonminimal}}$$

$$\hat{\mathcal{A}}^{\mu} = \underbrace{\mathcal{A}^{(3)\mu} + \mathcal{A}^{(4)\mu\nu}(\mathrm{i}\partial_{\nu})}_{\text{Minimal}} + \underbrace{\mathcal{A}^{(5)\mu\nu\alpha}(\mathrm{i}\partial_{\nu})(\mathrm{i}\partial_{\alpha}) + \ldots}_{\text{Nonminimal}}$$

[V.A. Kostelecký, M. Mewes, Phys. Rev. D **88**, 096006 (2013)]

Single-Fermion Sector

- It is always convenient to rewrite the latter Lagrangian in a more suggestive form:

$$\mathcal{L} = \frac{1}{2}\bar{\psi}\left(\hat{\Gamma}^{\nu}\mathrm{i}\partial_{\nu} - \hat{M}\right)\psi + \mathrm{H.c.}$$

- Using the 16 bilinear covariants, $\hat{\Gamma}^{\mu}$ and $\hat{M}$ can be decomposed as follows:

$$\begin{aligned} \hat{\Gamma}^{\nu} &= \gamma^{\nu} + \hat{c}^{\mu\nu}\gamma_{\mu} + \hat{d}^{\mu\nu}\gamma_5\gamma_{\mu} + \hat{e}^{\nu} + \mathrm{i}\hat{f}^{\nu}\gamma_5 + \frac{1}{2}\hat{g}^{\lambda\kappa\nu}\sigma_{\lambda\kappa}, \\ \hat{M} &= m_{\psi} + \hat{m} + \mathrm{i}\hat{m}_5\gamma_5 + \hat{a}^{\mu}\gamma_{\mu} + \hat{b}^{\mu}\gamma_5\gamma_{\mu} + \frac{1}{2}\hat{H}^{\mu\nu}\sigma_{\mu\nu}. \end{aligned}$$

Single-Fermion Sector

- The correspondences between the above coefficients are:

$$\hat{\mathcal{S}} = \hat{e} - \hat{m}, \quad \hat{\mathcal{P}} = \hat{f} - \hat{m}_5, \quad \hat{\mathcal{V}}^\mu = \hat{c}^\mu - \hat{a}^\mu,$$

$$\hat{\mathcal{A}}^\mu = \hat{d}^\mu - \hat{b}^\mu, \quad \hat{\mathcal{T}}^{\mu\nu} = \hat{g}^{\mu\nu} - \hat{H}^{\mu\nu}.$$

Operator	Type	d	CPT	Cartesian coefficients
$\hat{m}$	Scalar	Odd ≥ 5	Even	$m^{(d)\alpha_1\alpha_2\ldots\alpha_{d-3}}$
$\hat{m}_5$	Pseudoscalar	Odd ≥ 5	Even	$m_5^{(d)\alpha_1\alpha_2\ldots\alpha_{d-3}}$
$\hat{a}^\mu$	Vector	Odd ≥ 3	Odd	$a^{(d)\mu\alpha_1\alpha_2\ldots\alpha_{d-3}}$
$\hat{b}^\mu$	Pseudovector	Odd ≥ 3	Odd	$b^{(d)\mu\alpha_1\alpha_2\ldots\alpha_{d-3}}$
$\hat{c}^\mu$	Vector	Even ≥ 4	Even	$c^{(d)\mu\alpha_1\alpha_2\ldots\alpha_{d-3}}$
$\hat{d}^\mu$	Pseudovector	Even ≥ 4	Even	$d^{(d)\mu\alpha_1\alpha_2\ldots\alpha_{d-3}}$
$\hat{e}$	Scalar	Even ≥ 4	Odd	$e^{(d)\alpha_1\alpha_2\ldots\alpha_{d-3}}$
$\hat{f}$	Pseudoscalar	Even ≥ 4	Odd	$f^{(d)\alpha_1\alpha_2\ldots\alpha_{d-3}}$
$\hat{g}^{\mu\nu}$	Tensor	Even ≥ 4	Odd	$g^{(d)\mu\nu\alpha_1\alpha_2\ldots\alpha_{d-3}}$
$\hat{H}^{\mu\nu}$	Tensor	Odd ≥ 3	Even	$H^{(d)\mu\nu\alpha_1\alpha_2\ldots\alpha_{d-3}}$

[Kostelecký, Mewes (2013), arXiv:1308.4973]

Electromagnetic Sector

Lagrange density for the electromagnetic sector

$$\mathcal{L}_{(1+3)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\varepsilon^{\lambda\kappa\mu\nu}A_\lambda(\hat{k}_{AF})_\kappa F_{\mu\nu} - \frac{1}{4}F_{\kappa\lambda}(\hat{k}_F)^{\kappa\lambda\mu\nu}F_{\mu\nu}\,,$$

where A_μ is the $U(1)$ gauge field and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the field strength

Background fields

$$(\hat{k}_{AF})_\kappa = \sum_{\mathrm{d=odd}} (k_{AF}^{(d)})_\kappa{}^{\alpha_1\ldots\alpha_{(d-3)}}\partial_{\alpha_1}\ldots\partial_{\alpha_{(d-3)}}$$

$$(\hat{k}_F)^{\kappa\lambda\mu\nu} = \sum_{\mathrm{d=even}} (k_F^{(d)})^{\kappa\lambda\mu\nu\alpha_1\ldots\alpha_{(d-4)}}\partial_{\alpha_1}\ldots\partial_{\alpha_{(d-4)}}.$$

[V.A. Kostelecký and M. Mewes, Phys. Rev. D **80**, 015020 (2009)]

Neutrino Sector

Lagrange density for the neutrino sector

$$\mathcal{L} = \frac{1}{2}\bar{\Psi}_A\left(\gamma^\mu \mathrm{i}\partial_\mu \delta_{AB} - M_{AB} + \hat{\mathcal{Q}}_{AB}\right)\Psi_B + \mathrm{H.c.}$$

Euler-Lagrange equations

$$\left(\gamma^\mu \mathrm{i}\partial_\mu \delta_{AB} - M_{AB} + \hat{\mathcal{Q}}_{AB}\right)\Psi_B = 0.$$

Background fields

$$\begin{aligned}\hat{\mathcal{Q}}_{AB} &= \sum_I \hat{\mathcal{Q}}^I_{AB}\gamma_I, \\ &= \hat{\mathcal{S}}_{AB} + \mathrm{i}\hat{\mathcal{P}}_{AB}\gamma_5 + \hat{\mathcal{V}}^\mu_{AB}\gamma_\mu + \hat{\mathcal{A}}^\mu_{AB}\gamma_5\gamma_\mu + \frac{1}{2}\hat{\mathcal{T}}^{\mu\nu}_{AB}\sigma_{\mu\nu}.\end{aligned}$$

[V.A. Kostelecký and M. Mewes, Phys. Rev. D **85**, 096005 (2012)]

Neutrino Sector

A useful refinement first involves decomposing $\hat{\mathcal{Q}}_{AB}$ as:

$$\gamma^\mu p_\mu \delta_{AB} - M_{AB} + \hat{\mathcal{Q}}_{AB} = \hat{\Gamma}^\mu_{AB} p_\mu - \hat{M}_{AB},$$

in analogy to the usual decomposition in the single-fermion limit of the minimal SME.

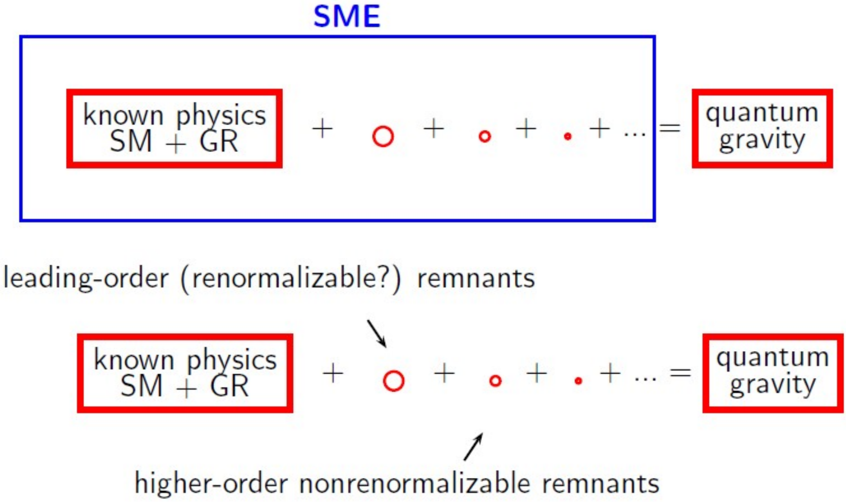
Splitting CPT-even and CPT-odd terms

$$\hat{\Gamma}^\mu = \gamma^\mu \delta_{AB} + \hat{c}^{\nu\mu}_{AB}\gamma_\nu + \hat{d}^{\nu\mu}_{AB}\gamma_5\gamma_\nu + \hat{e}^\mu_{AB} + \mathrm{i}\hat{f}^\mu_{AB}\gamma_5 + \frac{1}{2}\hat{g}^{\kappa\lambda\mu}_{AB}\sigma_{\kappa\lambda},$$

$$\hat{M} = M_{AB} + \hat{m}_{AB} + \mathrm{i}\hat{m}_{5AB}\gamma_5 + \hat{a}^\mu_{AB}\gamma_\mu + \hat{b}^\mu_{AB}\gamma_5\gamma_\mu + \frac{1}{2}\hat{H}^{\mu\nu}_{AB}\sigma_{\mu\nu},$$

where A and B range over $2N$ values. We must recall that we are allowing N Dirac neutrinos and N Majorana neutrinos.

Minimal and Nonminimal SME



[M. Mewes, Fourth SME Summer School 2021]

Nonminimal Planar Electrodynamics

Outline

Dimensional reduction of the electromagnetic sector

There are several procedures to derive a field theory of a (1+2)-dimensional electromagnetism from a (1+3)-dimensional parent theory.

- A first method could be a simple projection, i.e., to set the third component of the gauge field to zero and to disregard any dependence on the third spatial coordinate, e.g., $A^{\hat{\mu}}(t, \mathrm{x}^{(3)}) \mapsto A^{\mu}(t, \mathrm{x}^{(2)})$ with $\mu \in \{0 \ldots 2\}$.
- An alternative, more sophisticated approach to construct a (1+2)-dimensional daughter theory from a (1+3)-dimensional parent theory, is to disconnect the third component of $A^{\hat{\mu}}$ from the gauge field and to reinterpret it as a scalar field ϕ where the third spatial coordinate is again omitted.

[J.A.A.S. Reis, Manoel M. Ferreira Jr. and M. Schreck, Phys Rev D. **100**, 095026 (2019)]

Dimensional reduction of the electromagnetic sector

Dimensional Reduction of the fields

$$A^{\hat{\mu}\neq 3}(t,\mathrm{x}^{(3)}) \mapsto A^{\mu}(t,\mathrm{x}^{(2)})\,,$$

$$A_{\hat{\mu}\neq 3}(t,\mathrm{x}^{(3)}) \mapsto A_{\mu}(t,\mathrm{x}^{(2)})\,,$$

$$A^{\hat{3}}(t,\mathrm{x}^{(3)}) \mapsto \phi(t,\mathrm{x}^{(2)})\,,$$

$$A_{\hat{3}}(t,\mathrm{x}^{(3)}) \mapsto -\phi(t,\mathrm{x}^{(2)})\,.$$

This technique is sometimes called dimensional reduction in the literature.
[H. Belich Jr., M.M. Ferreira Jr., J.A. Helayël-Neto, and M.T.D. Orlando, Phys. Rev. D **67**, 125011 (2003); H. Belich Jr., M.M. Ferreira Jr., J.A. Helayël-Neto, and M.T.D. Orlando, Phys. Rev. D **68**, 025005 (2003).]

Dimensional reduction of the electromagnetic sector

Due to the presence of Lorentz violation, dimensional reduction must also be applied to the background fields and the Levi-Civita symbol:

Dimensional reduction of background

$$(\hat{k}_{AF})^{\hat{\kappa}\neq 3}(t,\mathrm{x}^{(3)}) \mapsto (\hat{k}_{AF})^{\kappa}(t,\mathrm{x}^{(2)}),$$

$$(\hat{k}_{AF})^{\hat{3}}(t,\mathrm{x}^{(3)}) \mapsto \hat{k}_{AF}(t,\mathrm{x}^{(2)}),$$

$$(\hat{k}_{AF})_{\hat{\kappa}\neq 3}(t,\mathrm{x}^{(3)}) \mapsto (\hat{k}_{AF})_{\kappa}(t,\mathrm{x}^{(2)}),$$

$$(\hat{k}_{AF})_{\hat{3}}(t,\mathrm{x}^{(3)}) \mapsto -\hat{k}_{AF}(t,\mathrm{x}^{(2)}),$$

$$\varepsilon^{\lambda\mu\nu 3} \mapsto \varepsilon^{\lambda\mu\nu},$$

where $\varepsilon^{\lambda\mu\nu}$ is the Levi-Civita symbol in (1+2) dimensions.

Dimensional reduction of electromagnetic sector

Carrying out this procedure for the individual terms, we get:

$$-\frac{1}{4}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}} \mapsto -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\,,$$

$$-\frac{1}{4}F_{\hat{\kappa}\hat{\lambda}}(\hat{k}_F)^{\hat{\kappa}\hat{\lambda}\hat{\mu}\hat{\nu}}F_{\hat{\mu}\hat{\nu}} \mapsto -\frac{1}{4}F_{\kappa\lambda}(\hat{k}_F)^{\kappa\lambda\mu\nu}F_{\mu\nu} - \partial_\kappa\phi(\hat{k}_\phi)^{\kappa\mu}\partial_\mu\phi + F_{\kappa\lambda}(\hat{k}_{\phi F})^{\kappa\lambda\mu}\partial_\mu\phi\,,$$

$$\frac{1}{2}\varepsilon^{\hat{\lambda}\hat{\kappa}\hat{\mu}\hat{\nu}}A_{\hat{\lambda}}(\hat{k}_{AF})_{\hat{\kappa}}F_{\hat{\mu}\hat{\nu}} \mapsto -\varepsilon^{\lambda\kappa\mu}A_\lambda(\hat{k}_{AF})_\kappa\partial_\mu\phi - \frac{1}{2}\varepsilon^{\lambda\mu\nu}A_\lambda(\hat{k}_{AF})F_{\mu\nu} - \varepsilon^{\mu\kappa\nu}\phi(\hat{k}_{AF})_\kappa\partial_\mu A_\nu\,,$$

where we have defined $(\hat{k}_\phi)^{\kappa\mu} \equiv (\hat{k}_F)^{\kappa 3\mu 3}$ and $(\hat{k}_{\phi F})^{\kappa\lambda\mu} \equiv (\hat{k}_F)^{\kappa\lambda\mu 3}$.

Dimensional reduction of the electromagnetic sector

The planar Lagrange density obtained after dimensional reduction is

$$\begin{aligned}\mathcal{L}_{(1+2)} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\varepsilon^{\lambda\mu\nu}A_\lambda(\hat{k}_{AF})F_{\mu\nu} \\ &- \frac{1}{4}F_{\kappa\lambda}(\hat{k}_F)^{\kappa\lambda\mu\nu}F_{\mu\nu} - \partial_\kappa\phi(\hat{k}_\phi)^{\kappa\mu}\partial_\mu\phi \\ &+ \varepsilon^{\nu\kappa\mu}\left[\phi(\hat{k}_{AF})_\kappa\partial_\mu A_\nu - A_\nu(\hat{k}_{AF})_\kappa\partial_\mu\phi\right] \\ &+ F_{\kappa\lambda}(\hat{k}_{\phi F})^{\kappa\lambda\mu}\partial_\mu\phi\,.\end{aligned}$$

- The first term describes the kinematics of an eletromagnetism in (1+2) spacetime dimensions;
- The second is the kinematic term of the scalar field;
- The third and fourth are the direct successors of the *CPT*-odd and *CPT*-even modifications in (1+3) dimensions.

General Euler-Lagrange Equations

Euler-Lagrange equation for higher-derivative field theories:

$$0 = \frac{\partial \mathcal{L}}{\partial \psi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \right) + \partial_\mu \partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \psi)} \right) - \cdots + (-1)^n \partial_{\mu_1} \ldots \partial_{\mu_n} \left(\frac{\partial \mathcal{L}}{\partial(\partial_{\mu_1} \ldots \partial_{\mu_n} \psi)} \right) .$$

$$0 = \Box \phi - 2(\hat{k}_\phi)^{\kappa\mu} \partial_k \partial_\mu \phi - \varepsilon^{\kappa\mu\nu} (\hat{k}_{AF})_\kappa F_{\mu\nu} + (\hat{k}_{\phi F})^{\mu\kappa\lambda} \partial_\mu F_{\kappa\lambda} ,$$

$$0 = \partial_\nu F^{\mu\nu} - \varepsilon^{\mu\nu\rho} (\hat{k}_{AF}) F_{\nu\rho} + (\hat{k}_F)^{\mu\sigma\nu\rho} \partial_\sigma F_{\nu\rho} - 2\varepsilon^{\mu\nu\rho} (\hat{k}_{AF})_\nu \partial_\rho \phi + 2(\hat{k}_{\phi F})^{\mu\nu\rho} \partial_\nu \partial_\rho \phi .$$

Gauge Fixing

- Gauge invariance prohibits a perturbative treatment of the daughter theory in (1+2) dimensions.
- Therefore, we add a gauge-fixing term with the gauge-fixing parameter ξ. Doing so, we get

$$\mathcal{L}^{\mathrm{gf}}_{(1+2)} \equiv \mathcal{L}_{(1+2)} - \frac{1}{2\xi}(\partial \cdot A)^2 \,.$$

- We transform the new Lagrange density to momentum space where it can be written in a suggestive form as follows:

$$\mathcal{L}^{\mathrm{gf}}_{(1+2)} = \frac{1}{2}(A, \phi) \begin{pmatrix} \hat{M} & \hat{U} - \mathrm{i}\hat{V} \\ (\hat{U} + \mathrm{i}\hat{V})^T & \hat{S} \end{pmatrix} \begin{pmatrix} A \\ \phi \end{pmatrix} \,.$$

Convenient Formulation of Theory

In terms of the (3×3) matrix

$$\hat{M}_{\mu\nu} = -p^2\Theta_{\mu\nu} + \hat{K}_{\mu\nu} + \mathrm{i}\hat{L}_{\mu\nu} - \frac{p^2}{\xi}\Omega^{\mu\nu} ,$$

the scalar

$$\hat{S} = p^2 - \hat{D} ,$$

the projectors

$$\Theta_{\mu\nu} \equiv \eta_{\mu\nu} - \Omega_{\mu\nu} ,$$

$$\Omega_{\mu\nu} \equiv \frac{p_\mu p_\nu}{p^2} .$$

and the Lorentz-violating operators

$$\hat{K}_{\mu\nu} \equiv 2(\hat{k}_F)_{\mu\kappa\beta\nu} p^\kappa p^\beta ,$$

$$\hat{L}_{\mu\nu} \equiv 2(\hat{k}_{AF})\varepsilon_{\mu\beta\nu} p^\beta ,$$

$$\hat{U}_{\mu} \equiv 2(\hat{k}_{\phi F})_{\mu\kappa\beta} p^\kappa p^\beta ,$$

$$\hat{V}_{\mu} \equiv 2\varepsilon_{\mu\kappa\nu}(\hat{k}_{AF})^\kappa p^\nu ,$$

$$\hat{D} \equiv 2(\hat{k}_\phi)^{\kappa\mu} p_\kappa p_\mu .$$

Green's Function for Scalar Field

The Lagrange density for the scalar field reads

$$\mathcal{L}_\phi = \frac{1}{2}\phi\hat{S}\phi\,,$$

where

$$\hat{S} = p^2 - \hat{D}\,, \qquad \hat{D} = 2(\hat{k}_\phi)^{\kappa\mu} p_\kappa p_\mu$$

The Green's function corresponds to the inverse of the operator $\hat{S}$ whose result is readily obtained:

$$\Delta_\phi = \frac{1}{p^2 - \hat{D}}.$$

Green's Function for Planar Electromagnetic Field

The treatment of the planar electromagnetic field is a bit more involved. We consider the Lagrange density

$$\mathcal{L}_A = \frac{1}{2} A_\mu \hat{M}^{\mu\nu} A_\nu \, .$$

The inverse of the (3×3) matrix $\hat{M}^{\mu\nu}$ can be expressed in terms of the metric tensor and suitable contractions of the original matrix. We found

$$\Delta_{\mu\nu} = \frac{1}{\mathcal{R}} \left\{ \frac{1}{2} \left[(\hat{M}^{\alpha}{}_{\alpha})^2 - \hat{M}^{\alpha\beta} \hat{M}_{\beta\alpha} \right] \eta_{\mu\nu} - (\hat{M}^{\alpha}{}_{\alpha}) \hat{M}_{\mu\nu} + \hat{M}_{\mu\beta} \hat{M}^{\beta}{}_{\nu} \right\} ,$$

where the denominator $\mathcal{R}$ corresponds to

$$3!\mathcal{R} = (\hat{M}^{\alpha}{}_{\alpha})^3 - 3(\hat{M}^{\alpha\beta} \hat{M}_{\beta\alpha})(\hat{M}^{\gamma}{}_{\gamma}) + 2\hat{M}^{\alpha\beta} \hat{M}_{\beta\gamma} \hat{M}^{\gamma}{}_{\alpha} \, .$$

$$\hat{M}^{\alpha}{}_{\alpha} = -\left(2+\frac{1}{\xi}\right)p^2 + \hat{K}^{\alpha}{}_{\alpha}\,,$$

$$\hat{M}^{\alpha\beta}\hat{M}_{\beta\alpha} = \left(2+\frac{1}{\xi^2}\right)p^4 - 2p^2\hat{K}^{\alpha}{}_{\alpha} - \hat{L}^{\alpha\beta}\hat{L}_{\beta\alpha} + \hat{K}^{\alpha\beta}\hat{K}_{\beta\alpha}\,,$$

$$\begin{aligned}\hat{M}^{\alpha\beta}\hat{M}_{\beta\gamma}\hat{M}^{\gamma}{}_{\alpha} = &-\left(2+\frac{1}{\xi^3}\right)p^6 + 3p^4\hat{K}^{\alpha}{}_{\alpha} \\ &+ 3p^2(\hat{L}^{\alpha\beta}\hat{L}_{\beta\alpha} - \hat{K}^{\alpha\beta}\hat{K}_{\beta\alpha}) \\ &+ 3(\mathrm{i}\hat{L}^{\alpha\beta}\hat{K}_{\beta\gamma}\hat{K}^{\gamma}{}_{\alpha} - \hat{L}^{\alpha\beta}\hat{L}_{\beta\gamma}\hat{K}^{\gamma}{}_{\alpha}) \\ &+ \hat{K}^{\alpha\beta}\hat{K}_{\beta\gamma}\hat{K}^{\gamma}{}_{\alpha} - \mathrm{i}\hat{L}^{\alpha\beta}\hat{L}_{\beta\gamma}\hat{L}^{\gamma}{}_{\alpha}\,.\end{aligned}$$

Due to the tensor structure of $\Delta_{\mu\nu}$, we also need:

$$\hat{M}_{\mu\beta}\hat{M}^{\beta}{}_{\nu} = p^4\left(\Theta_{\mu\nu} + \frac{1}{\xi^2}\Omega_{\mu\nu}\right) - 2p^2(\hat{K}+\mathrm{i}\hat{L})_{\mu\nu} + (\hat{K}+\mathrm{i}\hat{L})_{\mu\beta}(\hat{K}+\mathrm{i}\hat{L})^{\beta}{}_{\nu}\,.$$

Modified Dispersion Relations: Scalar Field

At leading order in the operator $\hat{k}_\phi$, the positive-energy solutions are given by

$$E^{(\pm)}(\mathrm{p}) = \frac{-2\hat{k}_\phi^{0i} p^i \pm \Psi(\hat{k}_\phi)}{1 - 2\hat{k}_\phi^{00}}\Bigg|_{p_0 = \omega_0(\mathrm{p})} + \dots ,$$

$$\Psi(\hat{k}_\phi) = \sqrt{4(\hat{k}_\phi^{0i} p^i)^2 + (1 - 2\hat{k}_\phi^{00})(\mathrm{p}^2 + 2\hat{k}_\phi^{ij} p^i p^j)} ,$$

where all additional p_0 are understood to be replaced by the standard massless dispersion relation $\omega_0(\mathrm{p}) \equiv |\mathrm{p}|$.

Modified Dispersion Relations: Electromagnetic Field

Inserting the background fields, the denominator $\mathcal{R}$ can be written in the form

$$\begin{aligned}\mathcal{R} &= -\frac{p^2}{\xi}\left\{p^2(p^2-\hat{K}^{\alpha}{}_{\alpha})-\frac{1}{2}\left[\hat{K}^{\alpha\beta}\hat{K}_{\beta\alpha}-(\hat{K}^{\alpha}{}_{\alpha})^2-\hat{L}^{\alpha\beta}\hat{L}_{\beta\alpha}\right]\right\}\\ &= -\frac{p^4}{\xi}\mathcal{R}^{\text{phys}}\,,\end{aligned}$$

with

$$\mathcal{R}^{\text{phys}} = p^2-\left(1+\frac{1}{2}(\hat{k}_F)^{\mu\nu}{}_{\mu\nu}\right)\hat{K}^{\alpha}{}_{\alpha}+\hat{K}^{\mu\nu}(\hat{k}_F)_{\mu\kappa\nu}{}^{\kappa}-4(\hat{k}_{AF})^2\,.$$

Modified Dispersion Relations: Electromagnetic Field

Independently of the form of the Lorentz-violating background field, the standard dispersion relation in two spatial dimensions is a two-fold zero of $\mathcal{R}$ with respect to p_0:

$$\omega^{(1,2)}(\mathrm{p}) = \omega_0 \,.$$

The third dispersion relation involves the Lorentz-violating operators. For the first case, we simply set $\hat{L}_{\mu\nu} = 0$ and obtain:

$$\omega^{(3)}(\mathrm{p})|_{k_{AF}=0} = \sqrt{\mathrm{p}^2 + \frac{1}{2}\left[\hat{K}^{\alpha}{}_{\alpha} + \Upsilon(\hat{K})\right]}\,\Bigg|_{p_0=\omega_0(\mathrm{p})} + \dots \,,$$

$$\Upsilon(\hat{K}) = \sqrt{2\hat{K}^{\alpha\beta}\hat{K}_{\beta\alpha} - (\hat{K}^{\alpha}{}_{\alpha})^2}\,.$$

For the second case, we insert $\hat{K}_{\mu\nu} = 0$, which leads to

$$\omega^{(3)}(\mathrm{p})|_{k_F=0} = \sqrt{\mathrm{p}^2 + 4(\hat{k}_{AF})^2}\,\Bigg|_{p_0=\omega_0(\mathrm{p})} + \dots \,.$$

Classical Solutions

- At this point we have the necessary tools ready to deal with the field equations.
- We consider the uncoupled equations that are obtained by setting the couplings equal to zero. Furthermore, we take inhomogeneities into account:

$$j(x) = \Box\phi - 2(\hat{k}_\phi)^{\kappa\mu}\partial_\kappa\partial_\mu\phi\,,$$

$$j^\mu(x) = \Box A^\mu + \varepsilon^{\mu\nu\varrho}\hat{k}_{AF}F_{\nu\varrho} - (\hat{k}_F)^{\mu\sigma\nu\varrho}\partial_\sigma F_{\nu\varrho}\,,$$

where we used the Lorenz gauge condition $\partial \cdot A = 0$ in the second equation to fix the gauge.

- The inhomogeneity associated with the scalar field is $j(x)$ and $j^\mu(x)$ is an external, conserved four-current density coupled to the electromagnetic field.

The general homogeneous solution is a superposition of plane-wave solutions involving the modified dispersion relations:

$$\phi^{\mathrm{hom}}(x) = \int \frac{\mathrm{d}^2 p}{(2\pi)^2} \sum_k \frac{1}{2E^{(k)}(\mathrm{p})} \phi^{(k)}(x)\,,$$

$$\phi^{(k)}(x) = a^{(k)}(p) \exp(-\mathrm{i} p^{(k)}_\alpha x^\alpha) + a^{(k)*}(p) \exp(\mathrm{i} p^{(k)}_\alpha x^\alpha)\,.$$

Here, $a^{(k)}$ is an appropriate plane-wave amplitude, $a^{(k)*}$ its complex conjugate, and $(p^{(k)\alpha}) = (E^{(k)}, \mathrm{p})$ with the appropriate dispersion relations. Note that all dispersion relations $E^{(k)}$ must be summed over. The inhomogeneous solution can be written as a contour integral in the complex p_0 plane:

$$\phi^{\mathrm{in}}(x) = \frac{1}{(2\pi)^3} \int_{C_E} \mathrm{d}p_0 \int \mathrm{d}^2 p\, \Delta_\phi(p) \tilde{j}(p) \exp(-\mathrm{i} p_\alpha x^\alpha)\,,$$

with the Green's function $\Delta_\phi(p)$, the Fourier-transformed inhomogeneity $\tilde{j}(p)$, and an appropriate contour C_E.

The treatment of the electromagnetic field is a bit more involved. The plane-wave homogeneous solutions involve the modified polarization vectors as wave amplitudes:

$$A_\mu^{\mathrm{hom}}(x) = \int \frac{\mathrm{d}^2 p}{(2\pi)^2} \sum_k \frac{1}{2\omega^{(k)}(\mathrm{p})} A_\mu^{(k)}(x)\,,$$

$$A_\mu^{(k)}(x) = \varepsilon_\mu^{(k)}(p) \exp(-\mathrm{i} p_\alpha^{(k)} x^\alpha) + \varepsilon_\mu^{(k)*}(p) \exp(\mathrm{i} p_\alpha^{(k)} x^\alpha)\,.$$

The inhomogeneous solution for an external, conserved current density j^μ is obtained by means of the Green's function $\Delta_{\mu\nu}$. Therefore, the inhomogeneous solution can also be written as a contour integral in the complex p_0 plane:

$$A_\mu^{\mathrm{in}}(x) = \frac{1}{(2\pi)^3} \int_{C_\omega} \mathrm{d}p_0 \int \mathrm{d}^2 p\, \Delta_{\mu\nu}^{\mathrm{phys}}(p) \tilde{j}^\nu(p) \exp(-\mathrm{i} p_\alpha x^\alpha)\,,$$

where $\tilde{j}^\mu$ is the Fourier-transformed three-current density and C_ω is an appropriate contour.

Final Remarks

Outline

Summary and outlook

- Lots of motivations justifies the investigation of possible deviations from Lorentz symmetry;
- SME provides an effective sub-Planckian framework to parameterize CPT and Lorentz violation;
- Lorentz violation becomes manifest as background field that couples to the physical fields;
- Nonminimal terms should be considered as corrections becoming dominant for increasing energies;
- Under consideration: fermion sector with both spin-degenerate and spin-nondegenerate operators, Fotons and Neutrino;
- All results can be used in applications of field-theoretical and phenomenological problems.

Summary and outlook

- Nonminimal framework for electromagnetism in (2+1)-dimensions was obtained.
- Nonminimal terms should be considered as corrections becoming dominant for increasing energies.
- The modified dispersion relations for electromagnetic waves were computed at leading order in Lorentz violation.
- Results can be used in applications of field-theoretical and phenomenological problems like:
  - Applications of this lower-dimensional modified electromagnetism to planar condensed-matter systems,
  - Study nonminimal effects in processes like: Cherenkov radiation in (2+1)-dimensions, scattering in (2+1)-dimensions, ...

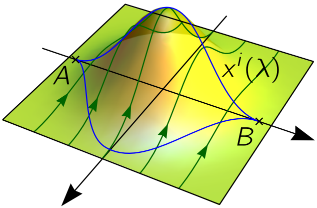
Summary and outlook

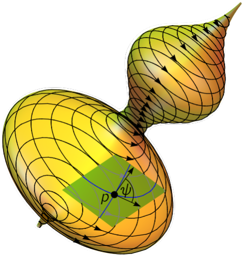
1. Planar Electrodynamics;
2. Generation of geometric phases in the nonminimal framework;
3. Fermions in (2+1)-dimension;
4. Thermodynamic properties of mesoscopic systems;
5. Correspondence between Classical Lagrangians and Finsler Geometry.

Summary and outlook

- Classical Lagrangians provide description of Lorentz violation for classical, pointlike particles based on SME;
- First-order Lagrangians for the whole nonminimal SME;
- What else could be done:
  - Promote the Lagrangians to Finsler geometries,
  - Calculate properties like: curvature, geodesics, connections, $\cdots$

    [Kostelecký, Phys. Lett. B **701**, 137 (2011)]





Acknowledgments

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Acknowledgments

Thank You!